University of Illinois at Chicago
Department of Mechanical & Industrial Engineering
ME 594 – Computational Compressible Flow

Project #2: **Godunov Method for 1D Inviscid Burgers Equation**

Due on November 23, 2015

This project deals with the solution of the 1D inviscid Burgers equation using the Godunov method described in Chapter 5 of Toro's book. Start from the source code `BUGOD.F` in the library *NUMERICA* that is available online.

1. Run the code for Test 3 in Chapter 5 and show that you obtain the same result as that in the book.

2. Modify subroutine `BCONDI` to implement Dirichlet type boundary conditions. Repeat Test 3 above using Dirichlet boundary conditions $\mathtt{U}(0) = -0.5$ and $\mathtt{U}(\mathtt{CELLS}+1) = 0$. Discuss the differences between the results of this part with those in part 1 above.

3. Obtain numerical results for the 1D Burgers equation with the following initial condition:

$$u(x,0) = \sin(2\pi x)\ , \quad 0 \le x \le 1 \tag{1}$$

   with periodic boundary conditions. Compare the results with the exact solution

$$u(x,t) = \sin\{2\pi[x - u(x,t)t]\} \tag{2}$$

   for different times until a shock is formed.

   NOTE: Equation (2) is implicit in $u(x,t)$ and must be solved iteratively. The Newton-Raphson method is recommended with initial guess taken from Eq. (1). Special treatment is needed for $x = 0, 0.5, 1$.

4. Discuss the effect of time step size (Δt) on the stability, and the effect of cell size (Δx) on the numerical diffusion for the problem in part 3 above.

5. Write a complete Technical Report for the project.